Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 6, 1999

FORMATION OF A PETAL-SHAPED STRUCTURE AT THE FRONT OF AN AXISYMMETRIC LIQUID FILM INDUCED BY COLLISION OF A DROP WITH A FLAT SURFACE

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UDC 621.793

A model for the formation of a petal-shaped structure is proposed that is based on Rayleigh–Taylor instability occurring at the stage of transition of supersonic flow to forced flow. At this stage, there is abrupt deceleration of the flow, reaching $(10^8$–$10^{10})g$. A dispersion relation is derived that allows one to obtain the length of the wave whose amplitude increases with maximum rate. The number of petals formed is determined assuming that this quantity is constant in time.

In the present paper, we consider the formation of a petal structure at the front of an initially axisymmetric radial jet induced by collision of a drop with a flat solid surface. This phenomenon has been noted by researchers from the beginning of studies of the problem of drop collisions with solid surfaces. However, at present, there is no qualitative theory of this process and the nature of instability leading to the formation of the petal structure is not yet understood. In the present paper, a detailed consideration of all collision stages shows that at the stage of transition from the "supersonic" to the stage of forced spreading, the liquid is subjected to large accelerations. This suggests that precisely Rayleigh–Taylor instability is responsible for the loss of axial symmetry and formation of petals.

Model of Collision. Let us consider a collision of a liquid spherical drop of radius r_0 with a flat rigid surface. Prior to the collision, the drop has velocity u_0 oriented along the normal to the surface. Introducing a coordinate system with origin at the frontal point, the z axis oriented toward the particle, and the r axis directed along the surface, we find the coordinate $r_c(t)$ of the intersection of the moving sphere with the plane $z = 0$. According to the results of [1], the drop velocity remains constant and equal to u_0 throughout the collision process. Using this fact, we obtain

$$r_c(t) = r_0\Big[1 - \Big(1 - \frac{u_0 t}{r_0}\Big)^2\Big]^{1/2}, \qquad \frac{dr_c}{dt} = u_c(t) = \frac{u_0(r_0 - u_0 t)}{[u_0 t(2r_0 - u_0 t)]^{1/2}}.$$

From the last expression it follows that there is the time interval $[0, t_1]$ in which the rate of displacement of the contact spot boundary exceeds the perturbation velocity c in the liquid. Therefore, within this time interval, the free boundary of the drop remains undeformed and radial flow does not develop.

We define t_1 as the time at which the rate of spread of the contact spot decreases to the perturbation velocity in the liquid c, i.e., $u_c(t_1) = c$. The solution of the last equation is easily obtained:

$$t_1 = \frac{r_0}{u_0}\Big(1 - \frac{1}{\sqrt{\mathrm{M}^2 + 1}}\Big)$$

($\mathrm{M} = u_0/c$ is the Mach number).

Kutateladze Institute of Thermal Physics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 6, pp. 91–96, November–December, 1999. Original article submitted July 14, 1998.

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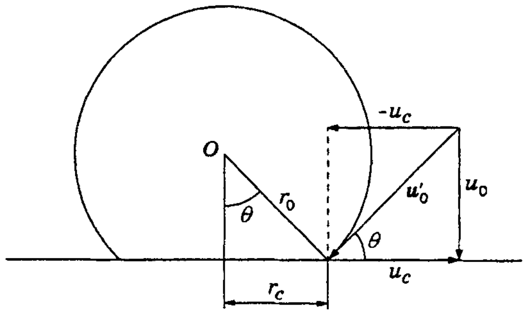

Fig. 1

In the cases considered here, the particle–surface collision underheating velocities are limited by Mach numbers $\mathrm{M} \ll 1$. Therefore, the previous expression can be written as

$$t_1 = \frac{r_0}{2u_0}\,\mathrm{M}^2. \tag{1}$$

From this moment, the shock-compressed region begins to unload by deformation of the free surface and radial flow develops whose initial velocity can be estimated as follows.

We consider a small vicinity of the contact line shown in Fig. 1. We go over to a movable coordinate system in which the contact spot boundary is immovable. In this coordinate system, the liquid velocity vector u_0' makes an angle θ with the solid surface. This corresponds to the problem of oblique collision of a jet with an obstacle [2], whose solution indicates that the radial flow velocity u_{r0}' coincides with u_0'.

As noted above, radial flow commences at the moment t_1 when the contact spot boundary velocity reaches the speed of sound in the liquid. Using Fig. 1, with allowance for (1), we find the contact spot radius $r_c(t_1)$, the angle $\theta(t_1)$, and u_0' corresponding to this moment of time t_1:

$$r_c(t_1) = r_0\mathrm{M}/\sqrt{1+\mathrm{M}^2} \sim r_0\mathrm{M}; \tag{2}$$

$$\theta(t_1) = \theta_1 = \arcsin\left(r_c(t_1)/r_0\right) \sim \mathrm{M}; \tag{3}$$

$$u_0' = u_0/\sin\theta_1. \tag{4}$$

Going over to a laboratory coordinate system, we obtain

$$u_{r0} = u_c + u_0/\sin\theta(t_1) = u_0(1+\cos\theta_1)/\sin\theta_1. \tag{5}$$

With allowance for (3) and the condition $\mathrm{M} \ll 1$, expression (5) takes the form

$$u_{r_0} = 2u_0/\mathrm{M} = 2c.$$

Thus, the initial velocity of the radial flow is equal to the speed of sound in the liquid. This conclusion is in good agreement with the results of [3].

From time t_1, unloading of the shock-compression region begins, and the pressure drops to the pressure of forced spreading [4]. At time $t_2 = 4r_0/c$, the forced spreading commences. In the time interval $(t_1,\, t_2)$, the velocity decreases from the speed of sound to about the initial collision velocity u_0. We estimate the characteristic average acceleration of the radial flow at the second stage:

$$\bar{a} \sim c^2/(4r_0). \tag{6}$$

Deriving formulas (6), we ignored the splash jet at the forced stage because of the smallness of the Mach number. For the characteristic parameters of our problem, this is a large quantity, and, hence, it is reasonable to assume that the formation of the petal structure is related to Rayleigh–Taylor instability.

Rayleigh–Taylor Instability. An analysis of the experimental data of [5, 6] shows that the number of petals increases with increase in the collision velocity, and a necessary condition for formation of the petal structure is the "collision" condition $t_i \ll t_{\rm osc}$. Here $t_i = 2r_0/u_0$ is the characteristic time of collision, $t_{\rm osc} = 2\pi\sqrt{\rho r_0^3/(8\sigma)}$ is the characteristic time of free oscillations of the drop, and ρ and σ are the density and surface tension of the drop material. Introducing the Weber number $\mathrm{We} = \rho r_0 u_0^2/\sigma$, we write the "collision" condition as

$$\sqrt{\mathrm{We}} \gg 1. \qquad (7)$$

When condition (7) is satisfied, the characteristic length of the perturbation is much shorter than the perimeter of the outer boundary (front) of the radial flow. Therefore, in a stability analysis, it is reasonable to use the approximation of a flat front. According to [7], we write the following equations of the linear hydrodynamics of an ideal incompressible liquid in a coordinate system attached to the film front (the unperturbed front of the film has the coordinate $y = 0$, and the y axis is directed to the depth of the liquid):

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \qquad \frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - a, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad (8)$$

Here a is the acceleration of the film front and u and v are the projections of the velocity vector onto the x and y axes, respectively.

Since there is no stationary motion in the chosen reference system, the perturbed fields of velocities and pressures can be written as

$$u = u', \qquad v = v', \qquad p = p_0 - \rho a y + p', \qquad (9)$$

where p_0 is the film front pressure (for $y = 0$); primes indicate perturbations of the fields of corresponding quantities. Thus, Eqs. (8) become

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}, \qquad \frac{\partial v'}{\partial t} = -\frac{1}{\rho}\frac{\partial p'}{\partial y}, \qquad \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0. \qquad (10)$$

The boundary condition follows from the absence of normal stresses at the front:

$$p - \sigma\frac{\partial^2 h'}{\partial x^2} = p_0, \qquad (11)$$

where h' is the instantaneous normal coordinate of the front. Here we ignore the air density since it is negligible compared to the liquid density. Taking into account the stationary pressure distribution (9), we write (11) for $y = h'$:

$$p' - \rho a h' - \sigma\frac{\partial^2 h'}{\partial x^2} = 0. \qquad (12)$$

The kinematic condition with accuracy up to first-order infinitesimal terms takes the form

$$v' = \frac{\partial h'}{\partial t}. \qquad (13)$$

We seek a solution of system (10), (12), and (13) in the form

$$\begin{bmatrix} u' \\ v' \\ p' \\ h' \end{bmatrix} = \begin{bmatrix} \tilde{u}(y) \\ \tilde{v}(y) \\ \tilde{p}(y) \\ \tilde{h} \end{bmatrix} \exp\,(ikx + \omega t). \qquad (14)$$

Here $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, and ω is the complex frequency of oscillations. For the chosen form of perturbations, instability of the boundary is indicated by the presence of a positive real term in the expression for ω.

Substituting (14) into (10) and (13), we obtain equations for the perturbation amplitudes. It should be noted that the kinematic and boundary conditions are used for $y = 0$ rather than for $y = h'$ because of the smallness of h'. Next, substituting the amplitudes of disturbances into (12), we obtain the following dispersion relation between the angular frequency ω and the wavenumber k:

$$\omega^2 = ak\Big(1 - \frac{\sigma k^2}{\rho a}\Big). \tag{15}$$

The length of the wave whose amplitude $k_{\max}$ increases with maximum rate is determined from the extremum condition for the function $\omega(k)$ (15):

$$\frac{d\omega}{dk}\Big|_{k=k_{\max}} = 0$$

or

$$k_{\max} = \sqrt{\rho a/(3\sigma)}, \qquad \lambda_{\max} = 2\pi\sqrt{3\sigma/(\rho a)}. \tag{16}$$

We introduce the quantity $N = 2\pi r_0/\lambda_{\max}$, which indicates how many wavelengths are present along the characteristic perimeter of the drop. Then, from (6) and (16) we obtain the following rough estimate for N:

$$N \sim \frac{1}{\mathrm{M}}\sqrt{\frac{\mathrm{We}}{12}}. \tag{17}$$

To determine the exact number of petals formed n, it is necessary to know the acceleration of the film front at the moment the stability is lost. Attempting to solve this problem, we assume that throughout the process of spreading, the number of petals formed at time t_1, remains unchanged i.e., $n = \mathrm{const}$. Using this condition, we can write

$$2\pi r(t_1) = n\lambda(t_1), \qquad 2\pi r(t) = n\lambda(t), \tag{18}$$

where $r(t)$ is the current radius of the front.

From Eqs. (18) it follows that at any time, an integer and time-independent number of waves is present at the front of the radial flow. From (18) we obtain

$$r(t) = \frac{r(t_1)}{\lambda(t_1)}\,\lambda(t). \tag{19}$$

Since a very short time interval (t_1, t_2) is considered, the front radius of the radial flow can be identified with the radius of the contact spot. In this case, we can write

$$r(t) = r_0\sqrt{2\frac{u_0}{r_0}\,t} \equiv r_0 f(t). \tag{20}$$

Using Eqs. (2), (16), and (20), we write Eq. (19) in the form

$$a(t) = a(t_1)\Big(\frac{r(t_1)}{r_0}\Big)^2\frac{1}{f^2(t)} = a(t_1)\mathrm{M}^2\frac{1}{f^2(t)}. \tag{21}$$

Using formula (21), we can express $a(t_1)$ in terms of the known average acceleration $\bar{a}$ (6):

$$\bar{a} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} a(t)\,dt \cong \frac{\mathrm{M}^2 a(t_1)}{t_2}\int_{t_1}^{t_2}\frac{dt}{f^2(t)} = a(t_1)\,\frac{\mathrm{M}}{8}\ln\frac{8}{\mathrm{M}},$$

or

$$a(t_1) \cong \frac{c^2/(4r_0)}{(\mathrm{M}/8)\ln(8/\mathrm{M})}. \tag{22}$$

Knowing the instantaneous acceleration (22), we obtain the number n of petals formed:

$$n = \frac{2\pi r(t_1)}{\lambda(t_1)} = \frac{r_0\mathrm{M}}{\sqrt{3\sigma/(\rho a(t_1))}} = \Big(\frac{\mathrm{We}}{12}\Big)^{1/2}\Big(\frac{\mathrm{M}}{8}\ln\frac{8}{\mathrm{M}}\Big)^{-1/2}. \tag{23}$$

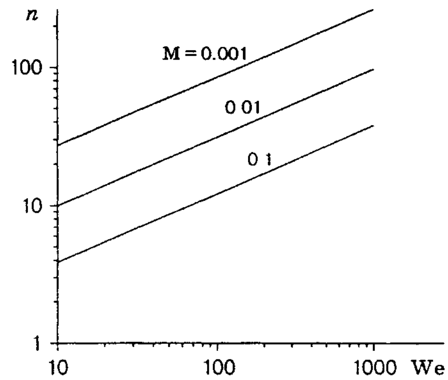

Fig. 2

Figure 2 shows the number of petals versus Weber numbers for typical Mach numbers. The theoretical results were tested using the experimental data obtained for collisions of melted metal drops with cool surfaces with total control of drop parameters (diameter, velocity, and temperature) immediately prior to collision [6]. Rapid solidification of drops in the process of spreading enabled us to record the shape of the petal structure. It is interesting to note that the experimental data obtained for numbers We $\sim$ 100 and M $\sim$ 0.01 give $n = 26$ and a calculation using (23) gives $n = 32$.

The studies performed suggest that Rayleigh–Taylor instability leads to formation of a petal structure at the front of the radial flow induced by drop collision with a solid surface.

This work was performed within the framework of the INTAS Project (No. 94-672).

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